EXERCISES FOR SECTION 4.4

Graphical Reasoning **In Exercises 1–4, use a graphing utility to graph the integrand. Use the graph to determine whether the definite integral is positive, negative, or zero.**

1. $\int_0^{\pi} \frac{4}{x^2+1}\,dx$
2. $\int_0^{\pi} \cos x\,dx$
3. $\int_{-2}^{2} x\sqrt{x^2+1}\,dx$
4. $\int_{-2}^{2} x\sqrt{2-x}\,dx$

In Exercises 5–26, evaluate the definite integral of the algebraic function. Use a graphing utility to verify your result.

5. $\int_0^1 2x\,dx$
6. $\int_2^7 3\,dv$
7. $\int_{-1}^0 (x-2)\,dx$
8. $\int_2^5 (-3v+4)\,dv$
9. $\int_{-1}^1 (t^2-2)\,dt$
10. $\int_1^3 (3x^2+5x-4)\,dx$
11. $\int_0^1 (2t-1)^2\,dt$
12. $\int_{-1}^1 (t^3-9t)\,dt$
13. $\int_1^2 \left(\frac{3}{x^2}-1\right)dx$
14. $\int_{-2}^{-1} \left(u-\frac{1}{u^2}\right)du$
15. $\int_1^4 \frac{u-2}{\sqrt{u}}\,du$
16. $\int_{-3}^3 v^{1/3}\,dv$
17. $\int_{-1}^1 (\sqrt[3]{t}-2)\,dt$
18. $\int_1^8 \sqrt{\frac{2}{x}}\,dx$
19. $\int_0^1 \frac{x-\sqrt{x}}{3}\,dx$
20. $\int_0^2 (2-t)\sqrt{t}\,dt$
21. $\int_{-1}^0 (t^{1/3}-t^{2/3})\,dt$
22. $\int_{-8}^{-1} \frac{x-x^2}{2\sqrt[3]{x}}\,dx$
23. $\int_0^3 |2x-3|\,dx$
24. $\int_1^4 (3-|x-3|)\,dx$
25. $\int_0^3 |x^2-4|\,dx$
26. $\int_0^4 |x^2-4x+3|\,dx$

In Exercises 27–32, evaluate the definite integral of the trigonometric function. Use a graphing utility to verify your result.

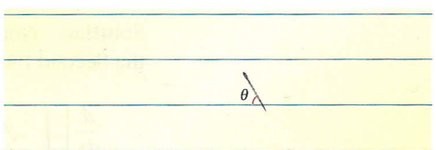
27. $\int_0^{\pi} (1+\sin x)\,dx$
28. $\int_0^{\pi/4} \frac{1-\sin^2\theta}{\cos^2\theta}\,d\theta$
29. $\int_{-\pi/6}^{\pi/6} \sec^2 x\,dx$
30. $\int_{\pi/4}^{\pi/2} (2-\csc^2 x)\,dx$
31. $\int_{-\pi/3}^{\pi/3} 4\sec\theta\tan\theta\,d\theta$
32. $\int_{-\pi/2}^{\pi/2} (2t+\cos t)\,dt$

33. ***Depreciation*** A company purchases a new machine for which the rate of depreciation is $dV/dt = 10{,}000(t-6)$, $0 \le t \le 5$, where V is the value of the machine after t years. Set up and evaluate the definite integral that yields the total loss of value of the machine over the first 3 years.

34. ***Buffon's Needle Experiment*** A horizontal plane is ruled with parallel lines 2 inches apart. If a 2-inch needle is tossed randomly onto the plane, the probability that the needle will touch a line is

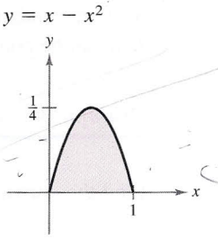
$$P = \frac{2}{\pi}\int_0^{\pi/2} \sin\theta\,d\theta$$

where θ is the acute angle between the needle and any one of the parallel lines. Find this probability.

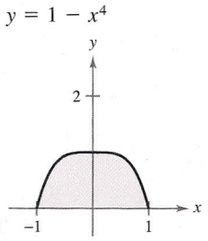


In Exercises 35–40, determine the area of the indicated region.

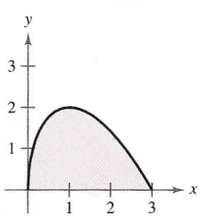
35. $y = x - x^2$



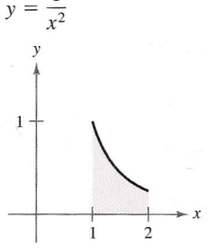
36. $y = 1 - x^4$



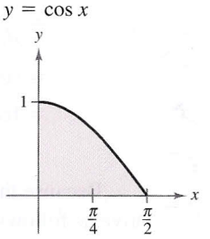
37. $y = (3-x)\sqrt{x}$



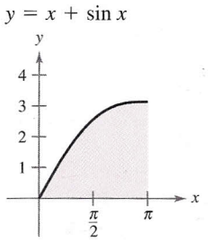
38. $y = \frac{1}{x^2}$



39. $y = \cos x$



40. $y = x + \sin x$



In Exercises 41–44, find the area of the region bounded by the graphs of the equations.

41. $y = 3x^2 + 1$, $x = 0$, $x = 2$, $y = 0$
42. $y = 1 + \sqrt[3]{x}$, $x = 0$, $x = 8$, $y = 0$
43. $y = x^3 + x$, $x = 2$, $y = 0$
44. $y = -x^2 + 3x$, $y = 0$